<u>Magnon spin transport driven by the spin</u> <u>chemical potential in Permalloy</u>

Hanfeng Wang EECS

Zhongqiang Hu EECS Dimple Kochar EECS

Introduction & Motivation





- Spintronics: spin degrees of freedom of electrons
- Precise addressing, low energy consumption, and non-volatility
- Magnons: quantum excitation of spin waves

The exploration of spin transport properties is of great importance!



- Injector: converting \boldsymbol{j}_{c} to \boldsymbol{j}_{s}
- Permalloy: spin current transport
- Detector: converting \mathbf{j}_{s} to \mathbf{j}_{c}

$$\frac{2e}{\hbar}\boldsymbol{j}_{\mathrm{s}} = -\frac{\nabla\mu_{\mathrm{s}}}{\rho_{\mathrm{e}}} \qquad \nabla^{2}\mu_{\mathrm{s}} = \frac{\mu_{\mathrm{s}}}{\ell_{\mathrm{sf}}^{2}} - \tau_{\mathrm{sf}}\frac{\partial}{\partial t}\nabla^{2}\mu_{\mathrm{s}}$$

[1] Nat. Phys. 11, 453–461 (2015)

Model Formulation



• Discretization (μ_i denotes the potential at the *i*th node):

$$C\left(\frac{d\mu_{\text{inj}}}{dt} - \frac{d\mu_{1}}{dt}\right) - C\left(\frac{d\mu_{1}}{dt} - \frac{d\mu_{2}}{dt}\right) = \frac{\mu_{1} - \mu_{2}}{R_{1}} - \frac{\mu_{\text{inj}} - \mu_{1}}{R_{1}} + \frac{\mu_{1}}{R_{2}} \qquad \text{for } i = 1$$

$$C\left(\frac{d\mu_{i-1}}{dt} - \frac{d\mu_{i}}{dt}\right) - C\left(\frac{d\mu_{i}}{dt} - \frac{d\mu_{i+1}}{dt}\right) = \frac{\mu_{i} - \mu_{i+1}}{R_{1}} - \frac{\mu_{i-1} - \mu_{i}}{R_{1}} + \frac{\mu_{i}}{R_{2}} \qquad \text{for } i \in \{2, 3, \dots, N-1\}$$

$$C\left(\frac{d\mu_{N-1}}{dt} - \frac{d\mu_{N}}{dt}\right) = -\frac{\mu_{N-1} - \mu_{N}}{R_{1}} + \frac{\mu_{N}}{R_{2}} \qquad \text{for } i = N$$
Note $C\left(\frac{R_{1}}{R_{1}} - \frac{R_{1}}{R_{1}}\right) = \frac{R_{1}}{R_{1}} - \frac{R_{2}}{R_{1}} \qquad \text{for } i = N$



• R_1 : spin transport; R_2 : spin leakage; C: spin accumulation

$$R_{1} = \frac{4e\rho_{e}\Delta L}{\hbar}, \qquad R_{2} = \frac{4e\rho_{e}\ell_{sf}^{2}}{\hbar\Delta L}, \qquad C = \frac{\hbar\tau_{sf}}{4e\rho_{e}\Delta L}$$

Methodology



Linear system: f(x(t), u(t)) = Ax(t) + Bu(t)

Simple Finite Difference Trapezoidal Time Integration Method is used: $\frac{dx}{dt} = f(x(t), u(t))$

$$x_j - x_{j-1} = \frac{\Delta \iota}{2} (f(x_j, u) + f(x_{j-1}, u))$$

• Fixed time step: $(I - \frac{\Delta t}{2}A)x_j = x_{j-1} + \frac{\Delta t}{2}(Ax_{j-1} + 2bu)$

 $\rightarrow JX = F$ with fixed $J \rightarrow LU$ decompose J, and use elimination at each time step

• Logarithmic time step: $(I - \frac{\Delta t_j}{2}A)x_j = x_{j-1} + \frac{\Delta t_j}{2}(Ax_{j-1} + 2bu)$

 \rightarrow solve using MATLAB's \ at every time step.

Demo of dynamical simulation in 2D case





| | R40 283.2 mΩ | R1 1,406 Q | R97 283.2 mΩ | R20 1.406 Ω | R98 283.2 mΩ | R24 1,406 Ω | | R26 | R100 283.2 mΩ |
|----------------|-------------------|-----------------|--------------------|-----------------------|------------------|--------------------------|------------------|------------------|------------------|
| | | C1 71.33 fF | | C10 71.33 fF | | C12 71.33 fF | | C13 71.33 fF | |
| R47 1.406 Ω | C26 71.33 fF | R42 1.406 Ω | C21 71.33 f | R43 F 1.406 Ω | C22 71,33 | R44 R44 IF 1.406 Ω | C23 71.33 | R45 < | C24 71.33 fF |
| | - | | | | + - | | | | |
| | • | R61 1.406 Ω | | R62 1.406 Ω C37 | + | R63 1.406 Ω | | R64 1.406 Ω | |
| | R105 283.2 mΩ | C36 71.33 fF | R104 283.2 mΩ | 71.33 fF | R103 283.2 mΩ | C38 71.33 fF | R102 283.2 mΩ | C39 71.33 fF | R101 |
| R69 1.406 Ω | C44 71.33 fF | R65 | C40 71.33 f | R66 F 1.406 Ω | C41 71.33 | R67 - | C42 71.33 | F 1.406 Ω < | C43 71.33 fF |
| | | | | | + | | | | |
| | + | R70 1.406 Ω | + + | R71 1.406 Ω | | R72 1.406 Ω | | R73 1.406 Ω | |
| | R110 283.12 mΩ | C45 71.33 fF | R109 283,2 mΩ | C46 71.33 fF | R108 283.2 mΩ | C47 71.33 fF | R107 283.2 mΩ | C48 71.33 fF | R106 283,2 mΩ |
| R78 1.406 Ω | C53 71.33 fF | R74 1.406 Ω | C49 71.33 1 | R75 F 1.406 Ω | C50 71.33 | R76 - | C51 71.33 | R77 < | C52 71.33 fF |
| | | | | | + | | | | |
| | • | R79 1.406 Ω | | R80 1.406 Ω | | R81 1.406 Ω | - | R82 1.406 Ω | - |
| | R115 283.2 mΩ | C54 71.33 fF | R114 283.2 mΩ | C55 71.33 fF | R113 283.2 mΩ | C56 71.33 fF | R112 283.2 mΩ | C57 71.33 fF | R111 283.2 mΩ |
| R87 | C62 71.33 fF | R83 1.406 Ω | C58 71.33 f | R84 F 1.406 Ω | C59 71,33 | R85 - | C60 71.33 | F 1.406 Ω < | C61 71.33 fF |
| | | | | | | | | | • |
| -(+) |) | R88 1.406 Ω | ~ | R89 1.406 Ω | | R90 • 1.406 Ω | | R91 • 1.406 Ω | |
| V1 8.7 mV | R120 283.2 mΩ | C63 71.33 fF | ₹ R119 283.2 mΩ | C64 71.33 fF | R118 283.2 mΩ | C65 71.33 fF | R117 283.2 mΩ | C66 71.33 fF | R116 283.2 mΩ |
| | | | | | | | | | |
| | | | | [h | ttps://ww | w.circuitla | ab.com/edi | tor/#?id= | 4vt65z22tg |



Verification using an online circuit simulator



 μ_{inj} = 8.7 µV, 400 nodes

 μ_{inj} = 8.7 mV, 25 nodes

[https://www.circuitlab.com/editor/#?id=4vt65z22tge9]

Model Order Reduction

MITEECS

Size = 900



Choose q=40 for 5% tolerence. Time 0.4%. Memory 1.0%

1D

Preconditioner



distribution of non-zero elements in A matrix





Model Order Reduction



Size = 900 Choose q=40 for 5% tolerence. Time 0.4%. Memory 1.0%

Size = 3600 Time ~90%. Memory ~3% (Maybe an another way is better)



1D Steady State and Parameter Analysis



- State vector $x = [\mu_1, \mu_2, ..., \mu_N]$, input $u = [\mu_{inj}]$, quantity of interest $y = \mu_{det} = \mu_N$
- The potential decays from the injector (μ_0) to the detector (μ_N)
- "Bad" solution: (1) decays too quickly, $\mu_{det} \approx 0$; (2) decays too slowly, $\mu_{det} \approx \mu_{inj}$
- 1st type bad solution, $\Delta \mu = \mu_{det} \mu_{inj}$ will have little change when parameters are perturbed in a small percentage (e.g., ~ 10%). This happens when $\ell_{sf} \ll L$:



1D Steady State and Parameter Analysis



• 2nd type bad solution: μ_{ave} has little change. This happens when $\ell_{sf} \gg L$:



• Only when $\ell_{sf} \sim L$, μ_{ave} and $\Delta \mu$ are both sensitive, and we get rid of both the "bad" solutions:

