Magnon spin transport driven by the spin chemical potential in Permalloy

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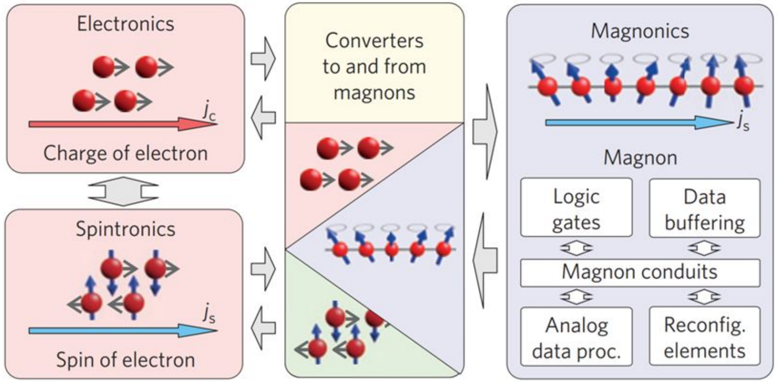
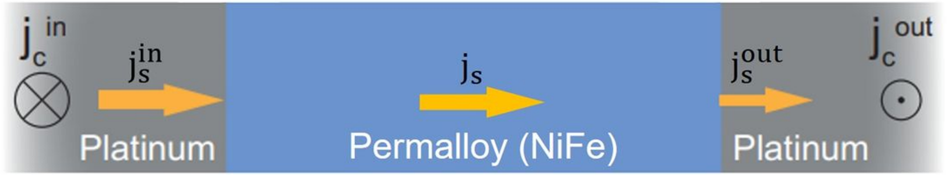
Introduction & Motivation



- Spintronics: spin degrees of freedom of electrons
- Precise addressing, low energy consumption, and non-volatility
- Magnons: quantum excitation of spin waves

The exploration of spin transport properties is of great importance!



- Injector: converting $\boldsymbol{j}_c$ to $\boldsymbol{j}_s$
- Permalloy: spin current transport
- Detector: converting $\boldsymbol{j}_s$ to $\boldsymbol{j}_c$

$$\frac{2e}{\hbar}\boldsymbol{j}_s = -\frac{\nabla\mu_s}{\rho_e} \qquad \nabla^2\mu_s = \frac{\mu_s}{\ell_{sf}^2} - \tau_{sf}\frac{\partial}{\partial t}\nabla^2\mu_s$$

[1] *Nat. Phys.* **11**, 453–461 (2015)

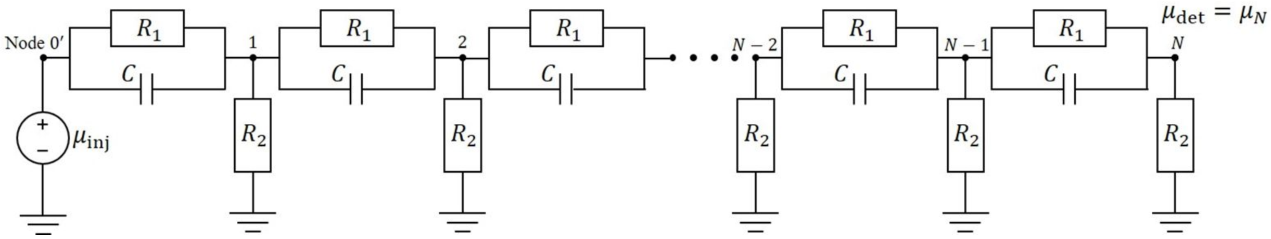
Model Formulation

- Discretization (μ_i denotes the potential at the ith node):

$$C\left(\frac{d\mu_{\text{inj}}}{dt}-\frac{d\mu_1}{dt}\right)-C\left(\frac{d\mu_1}{dt}-\frac{d\mu_2}{dt}\right)=\frac{\mu_1-\mu_2}{R_1}-\frac{\mu_{\text{inj}}-\mu_1}{R_1}+\frac{\mu_1}{R_2}\qquad \text{for } i=1$$

$$C\left(\frac{d\mu_{i-1}}{dt}-\frac{d\mu_i}{dt}\right)-C\left(\frac{d\mu_i}{dt}-\frac{d\mu_{i+1}}{dt}\right)=\frac{\mu_i-\mu_{i+1}}{R_1}-\frac{\mu_{i-1}-\mu_i}{R_1}+\frac{\mu_i}{R_2}\qquad \text{for i} \in \{2,3,\dots,N-1\}$$

$$C\left(\frac{d\mu_{N-1}}{dt}-\frac{d\mu_N}{dt}\right)=-\frac{\mu_{N-1}-\mu_N}{R_1}+\frac{\mu_N}{R_2}\qquad \text{for } i=N$$



- R_1: spin transport; R_2: spin leakage; C: spin accumulation

$$R_1=\frac{4e\rho_e\Delta L}{\hbar},\qquad R_2=\frac{4e\rho_e\ell_{\text{sf}}^2}{\hbar\Delta L},\qquad C=\frac{\hbar\tau_{\text{sf}}}{4e\rho_e\Delta L}$$

Methodology

Linear system: $f(x(t), u(t)) = Ax(t) + Bu(t)$

Simple Finite Difference Trapezoidal Time Integration Method is used: $\frac{dx}{dt} = f\big(x(t), u(t)\big)$

$$x_j - x_{j-1} = \frac{\Delta t}{2}(f(x_j, u) + f(x_{j-1}, u))$$

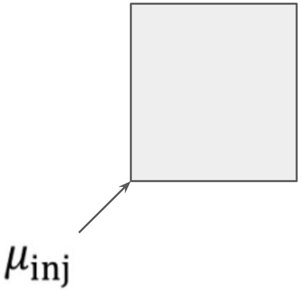
- Fixed time step: $(I - \frac{\Delta t}{2}A)x_j = x_{j-1} + \frac{\Delta t}{2}(Ax_{j-1} + 2bu)$

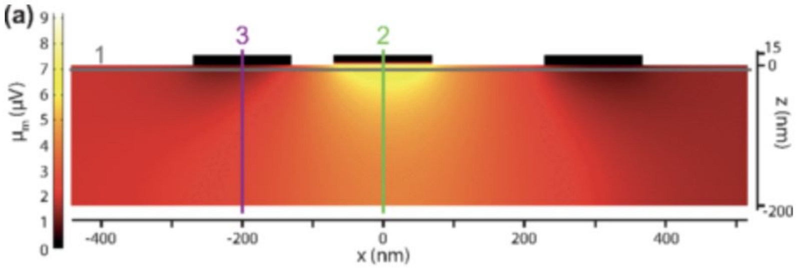
  → $JX = F$ with fixed J → LU decompose J, and use elimination at each time step

- Logarithmic time step: $(I - \frac{\Delta t_j}{2}A)x_j = x_{j-1} + \frac{\Delta t_j}{2}(Ax_{j-1} + 2bu)$

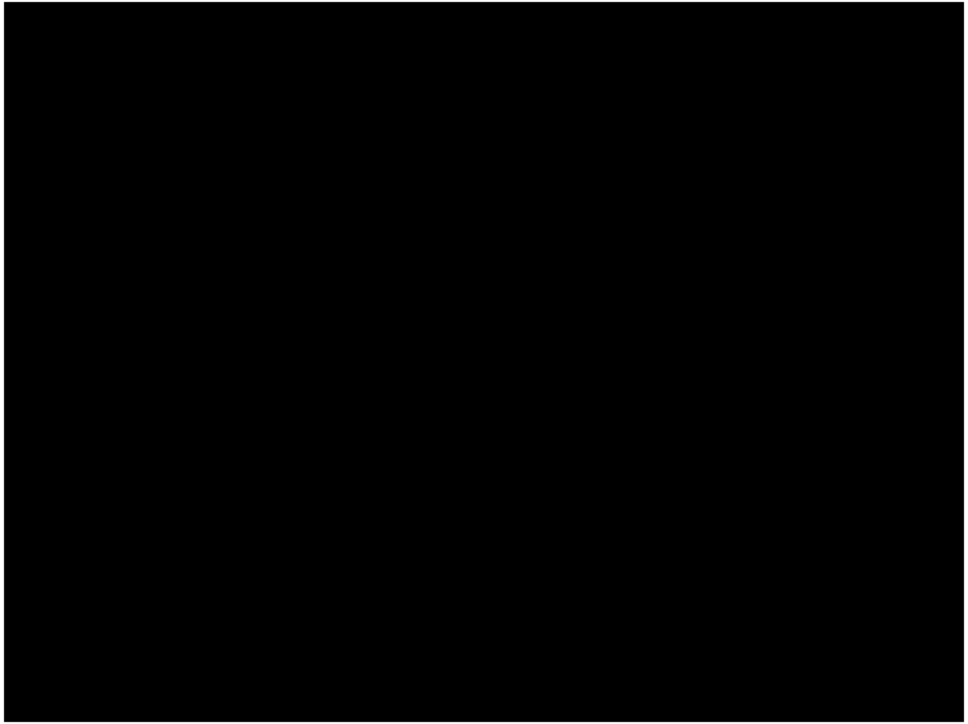
  → solve using MATLAB's \ at every time step.

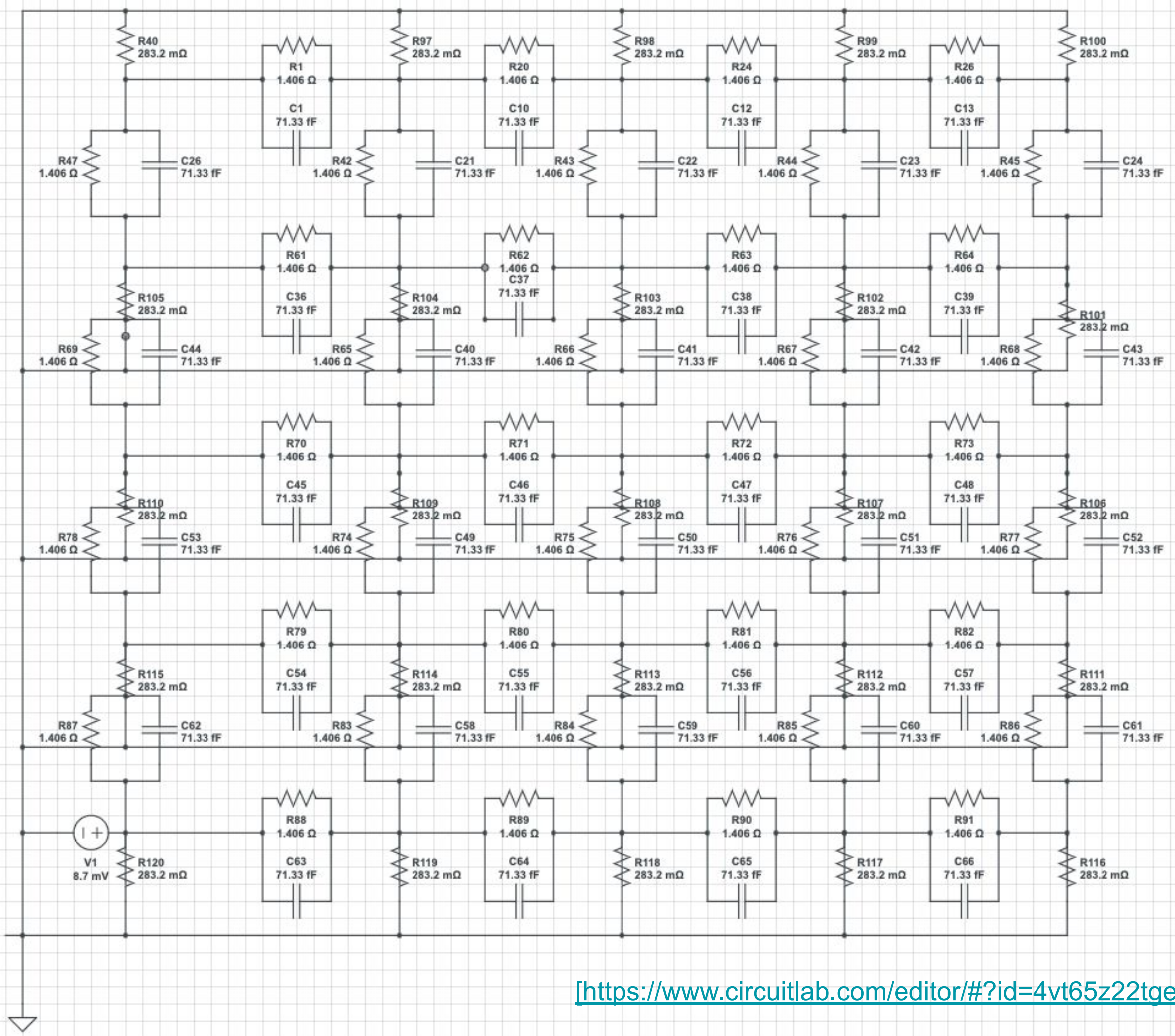
Demo of dynamical simulation in 2D case





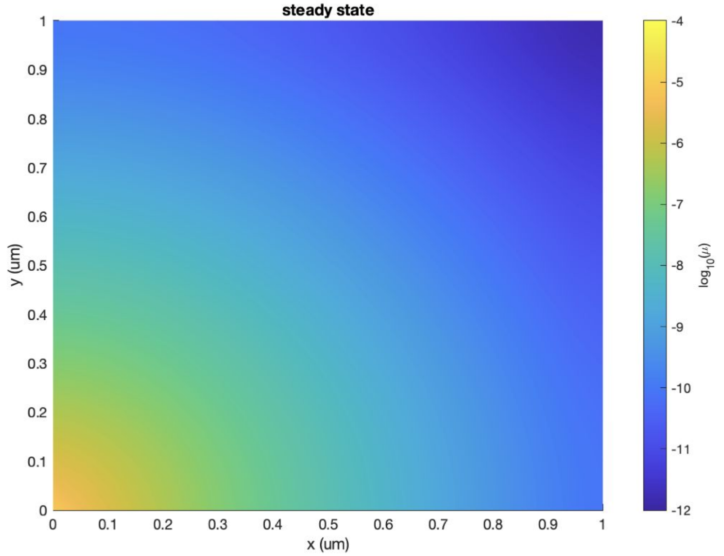
[Magnon spin transport driven by the magnon chemical potential in a magnetic insulator, L. J. Cornelissen, Phys. Rev. B **94**, 014412]



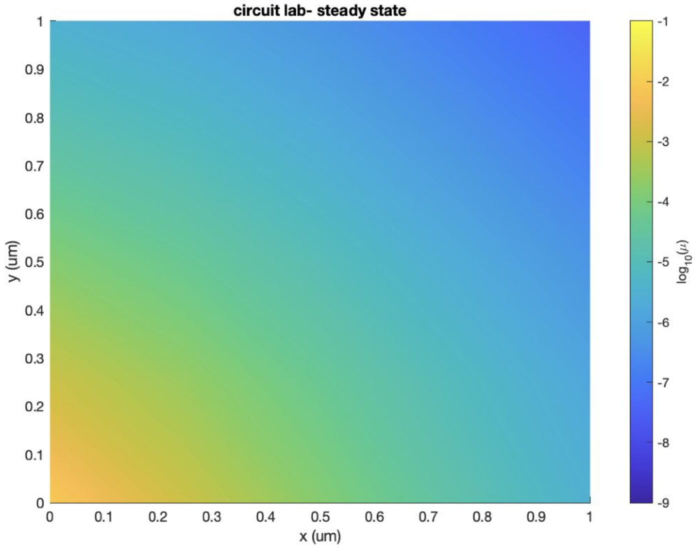


[https://www.circuitlab.com/editor/#?id=4vt65z22tge9]

Verification using an online circuit simulator



μ_{inj} = 8.7 μV, 400 nodes



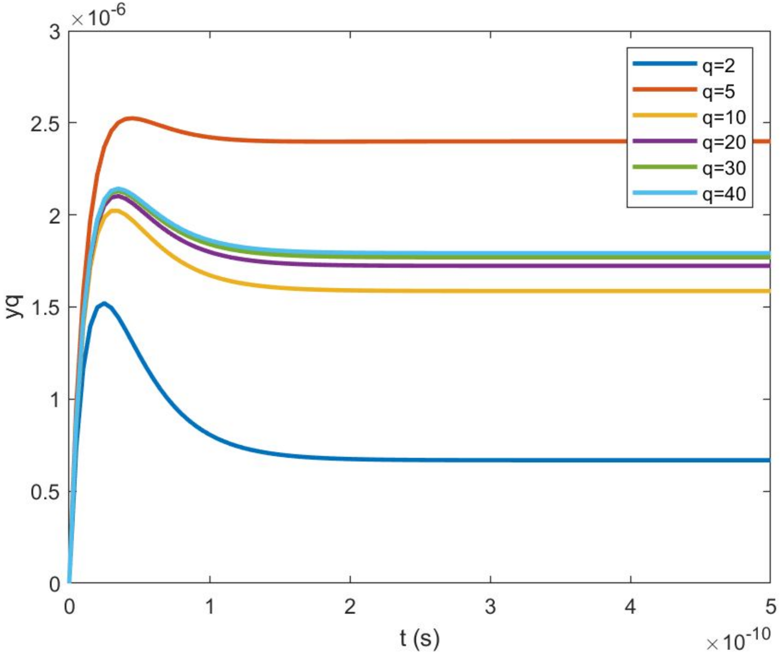
μ_{inj} = 8.7 mV, 25 nodes

[https://www.circuitlab.com/editor/#?id=4vt65z22tge9]

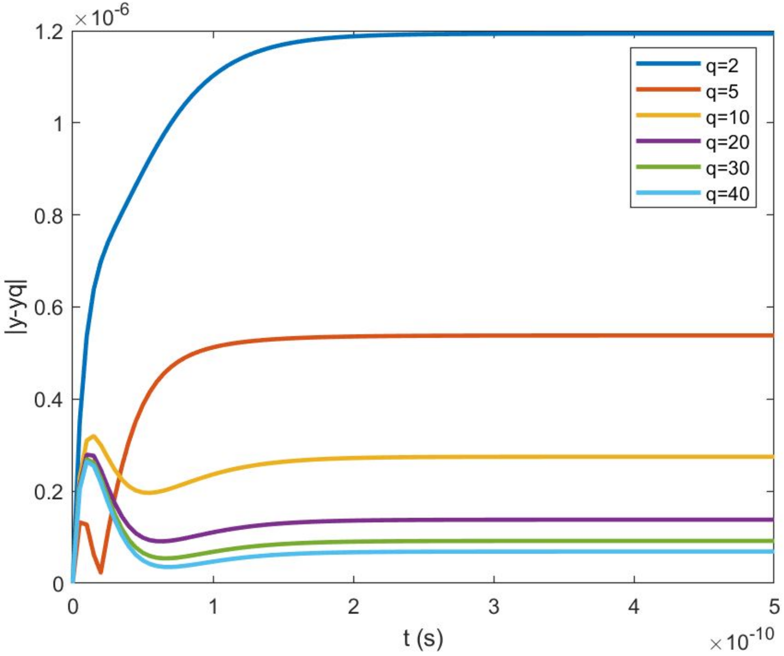
Model Order Reduction

Size = 900

Solution



Error

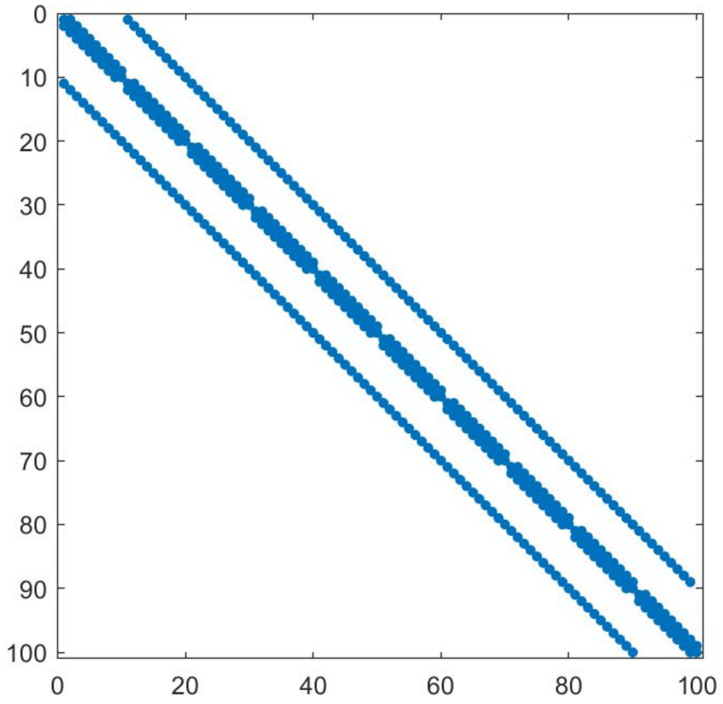


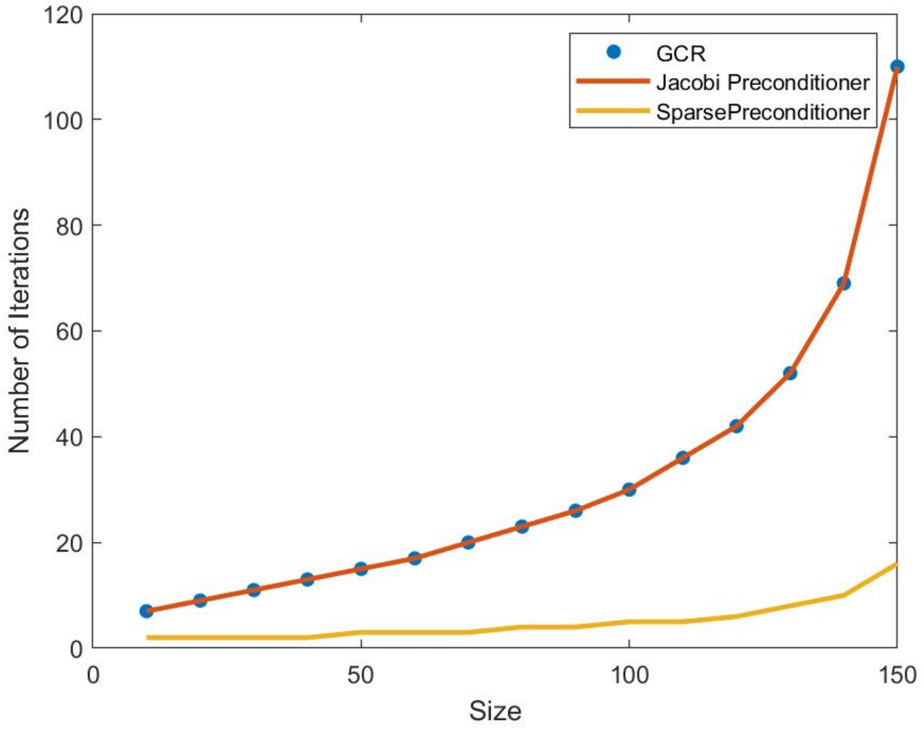
Choose q=40 for 5% tolerence. Time 0.4%. Memory 1.0%

1D

Preconditioner

distribution of non-zero elements in A matrix



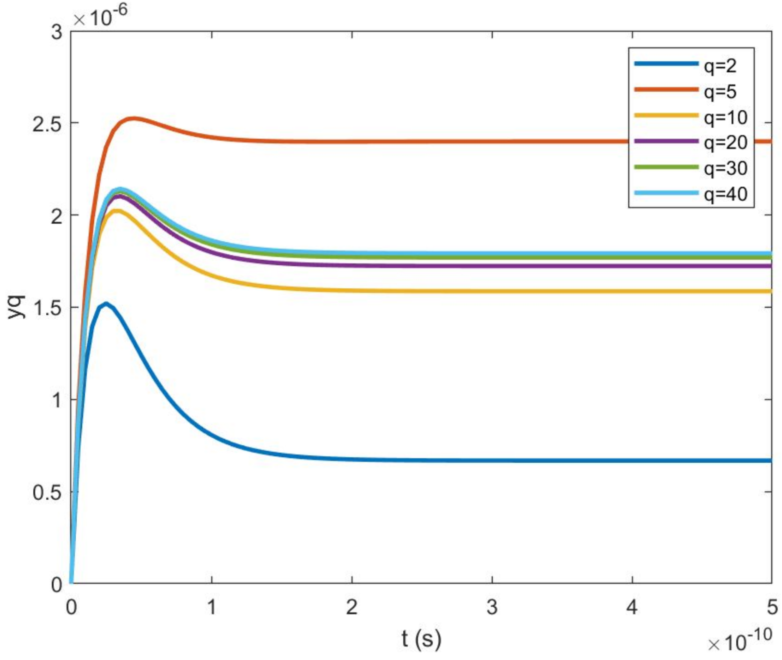


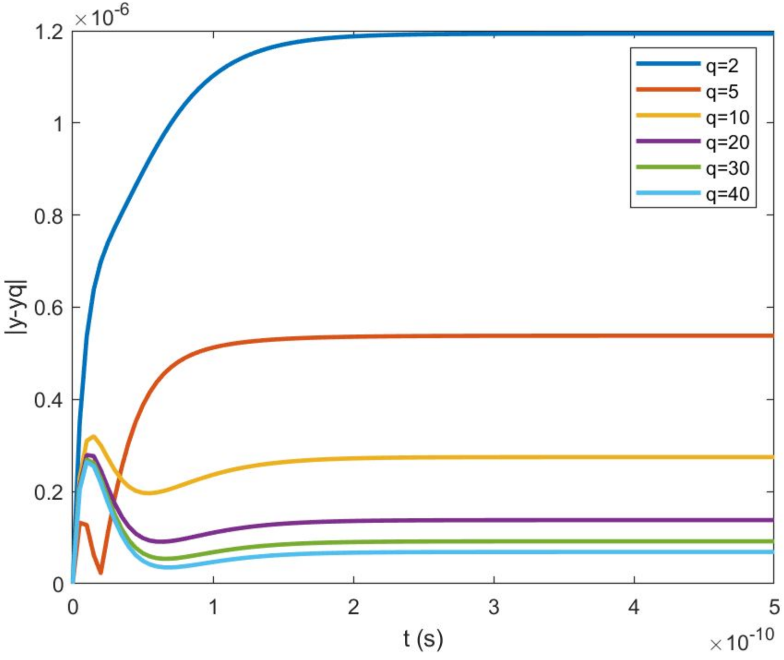
Thanks!

Model Order Reduction

Size = 900 Choose q=40 for 5% tolerence. Time 0.4%. Memory 1.0%

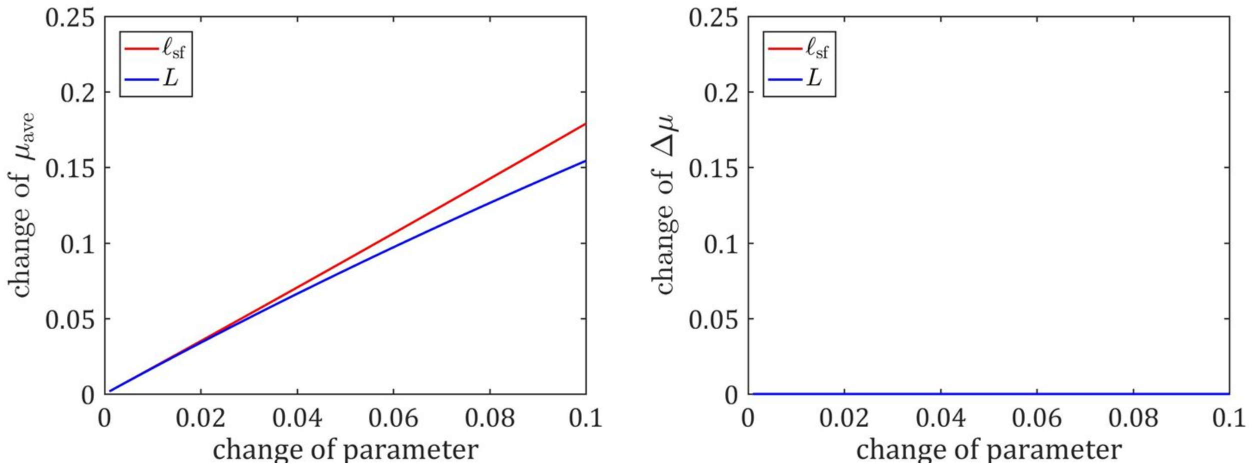
Size = 3600 Time ~90%. Memory ~3% (Maybe an another way is better)





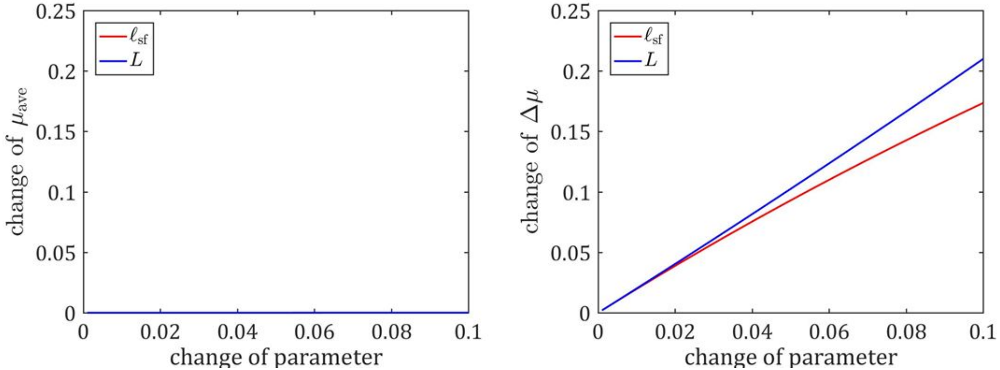
1D Steady State and Parameter Analysis

- State vector $x = [\mu_1, \ \mu_2, \ldots, \mu_N]$, input $u = [\mu_{\text{inj}}]$, quantity of interest $y = \mu_{\text{det}} = \mu_N$
- The potential decays from the injector (μ_0) to the detector (μ_N)
- "Bad" solution: (1) decays too quickly, $\mu_{\text{det}} \approx 0$; (2) decays too slowly, $\mu_{\text{det}} \approx \mu_{\text{inj}}$
- 1st type bad solution, $\Delta\mu = \mu_{\text{det}} - \mu_{\text{inj}}$ will have little change when parameters are perturbed in a small percentage (e.g., ~ 10%). This happens when $\ell_{\text{sf}} \ll L$:



1D Steady State and Parameter Analysis

- 2^{nd} type bad solution: μ_{ave} has little change. This happens when $\ell_{sf} \gg L$:



- Only when $\ell_{sf} \sim L$, μ_{ave} and $\Delta\mu$ are both sensitive, and we get rid of both the "bad" solutions: