Indian Institute of Technology, Bombay

Digital Signal Processing

EE - 338

Filter Design Assignment

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1 Student Details

Name : *Dimple Kochar*
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Filter Number : 79

2 Filter-1(Bandpass) Details

2.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 79
Since filter number is >75 , m = 79 - 75 = 4 and passband will be equiripple
q(m) = greatest integer strictly less than 0.1*m = greatest integer strictly less than 0.4 = 0
r(m) = m - 10*q(m) = 4 - 10*0 = 4
B_L(m) = 5 + 1.4*q(m) + 4*r(m) = 5 + 1.4*0 + 4*4 = 21
B_H(m) = B_L(m) + 10 = 21 + 10 = 31

We have to design a ***Band-Pass*** filter with passband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are as follows:

- **Passband** : 21 kHz to 31 kHz
- **Transition Band** : 2 kHz on either side of passband
- **Stopband** : 0-19 kHz and 33-160 kHz ($\because$ Sampling rate is 320 kHz)
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.2 Normalized Digital Filter Specifications

Sampling Rate = 320 kHz
Any frequency(Ω) upto 160 kHz($\frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as (since in the normalized frequency axis, sampling rate corresponds to 2π) :

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(SamplingRate)}$$

Therefore the corresponding normalized discrete filter specifications are:

- **Passband** : 0.13125π to 0.19375π
- **Transition Band** : 0.0125π on either side of passband
- **Stopband** : 0-0.11875π and 0.20625π-π
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.3 Analog filter specifications for Band-pass analog filter using Bilinear Transformation

The bilinear transformation is given as:

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get:

ω	Ω
0	0
0.11875π	0.1887
0.13125π	0.2091
0.19375π	0.3141
0.20625π	0.3358
π	∞

Therefore, the corresponding analog filter specifications using the bilinear transformation are:

- **Passband** : $0.2091(\Omega_{P_1})$ to $0.3141(\Omega_{P_2})$
- **Transition Band** : 0.1887 to 0.2091 & 0.3141 to 0.3358
- **Stopband** : 0 to $0.1887(\Omega_{S_1})$ and $0.3358(\Omega_{S_2})$ to ∞
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.4 Frequency Transformation & Relevant Parameters

We make use of the Bandpass transformation to transform the Band-Pass analog filter to a Lowpass analog filter. We require two parameters in such a case:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations:

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.2091 * 0.3141} = 0.2563$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 0.3141 - 0.2091 = 0.1050$$

Ω	Ω_L
0^+	$-\infty$
$0.1887(\Omega_{S_1})$	$-1.5181(\Omega_{L_{S_1}})$
$0.2091(\Omega_{P_1})$	$-1(\Omega_{L_{P_1}})$
$0.2563(\Omega_0)$	0
$0.3141(\Omega_{P_2})$	$1(\Omega_{L_{P_2}})$
$0.3358(\Omega_{S_2})$	$1.3356(\Omega_{L_{S_2}})$
∞	∞

2.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge** : 1 (Ω_{L_P})
- **Stopband Edge** : min($-\Omega_{L_{S_1}}$,$\Omega_{L_{S_2}}$) = min(1.5181, 1.3356) = 1.3356 (Ω_{L_S})
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Equiripple
- **Stopband Nature** : Monotonic

2.6 Analog Lowpass Transfer Function

We need an Analog Filter which has an *equiripple passband* and a *monotonic stopband*. Therefore, we design using the **Chebyshev** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now choosing the parameter ϵ of the Chebyshev filter to be $\sqrt{D_1}$, we get the min value of N as:

$$N_{min} = \lceil \frac{cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{cosh^{-1}(\frac{\Omega_{L_S}}{\Omega_{L_P}})} \rceil$$

$$N_{min} = \lceil 3.493 \rceil = 4$$

Now, the poles of the transfer function can be obtained by solving the equation:

$$1 + D_1 cosh^2(N_{min} cosh^{-1}(\frac{s}{j})) = 1 + 0.3841 cosh^2(4 cosh^{-1}(\frac{s}{j})) = 0$$

Solving for the roots (using MATLAB) we get:
Note that the above figure shows the poles of the Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$p_1 = -0.12216 - 0.96981\iota$$

$$p_2 = -0.12216 + 0.96981\iota$$

$$p_3 = -0.29493 + 0.40171\iota$$

$$p_4 = -0.29493 - 0.40171\iota$$

Using the above poles which are in the left half plane and the fact that N is even we can write the Analog Lowpass Transfer Function as:

$$H_{analog,LPF}(s_L) = \frac{(-1)^4 p_1 p_2 p_3 p_4}{\sqrt{(1+D_1)}(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)}$$

$$H_{analog,LPF}(s_L) = \frac{0.2017}{(s_L^2 + 0.24432 s_L + 0.95545)(s_L^2 + 0.58984 s_L + 0.24835)}$$

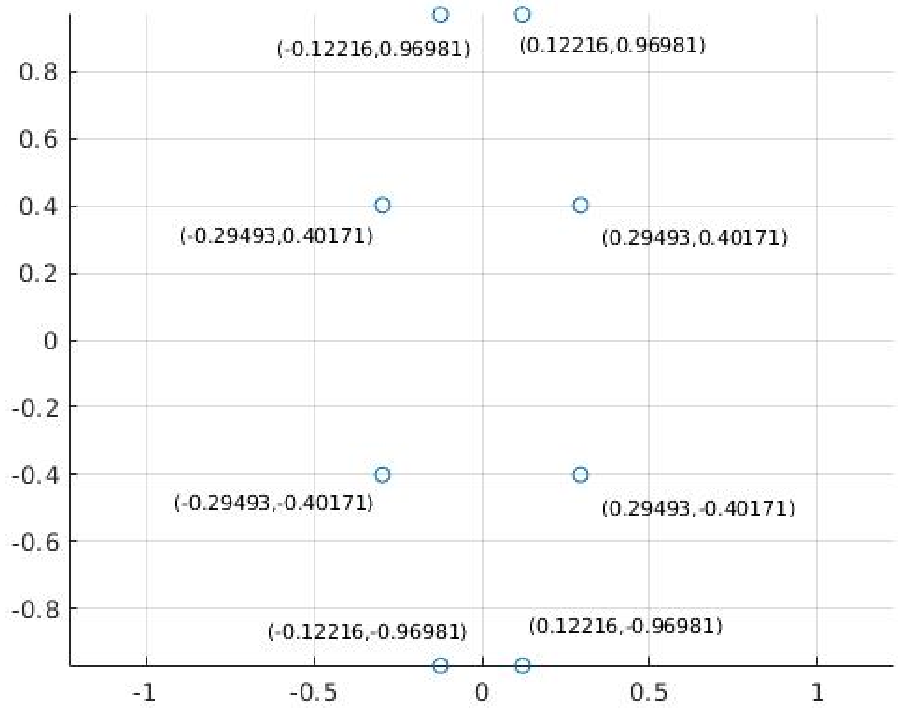

Figure 1: Poles of Magnitude Plot of Analog LPF

2.7 Analog Bandpass Transfer Function

The transformation equation is given by:

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values of the parameters B(0.1050) and Ω_0(0.2563), we get:

$$s_L = \frac{s^2 + 0.0657}{0.1050s}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BPF}(s)$ as:

$$\frac{0.2448 * 10^{-4} * s^4}{(s^8 + 0.0876s^7 + 0.2776s^6 + 0.0180s^5 + 0.0279s^4 + 0.0012s^3 + 0.0012s^2 + 2.482 * 10^{-5}s^1 + 1.862 * 10^{-5})}$$

2.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}(z)$ from $H_{analog,BPF}(s)$ as:

$$\frac{10^{-3} * (0.0173 - 0.0693z^{-2} + 0.1039z^{-4} - 0.0693z^{-6} + 0.0173z^{-8})}{1 - 6.8374z^{-1} + 21.3607z^{-2} - 39.6324z^{-3} + 47.6713z^{-4} - 38.0410z^{-5} + 19.6800z^{-6} - 6.0468z^{-7} + 0.8490z^{-8}}$$

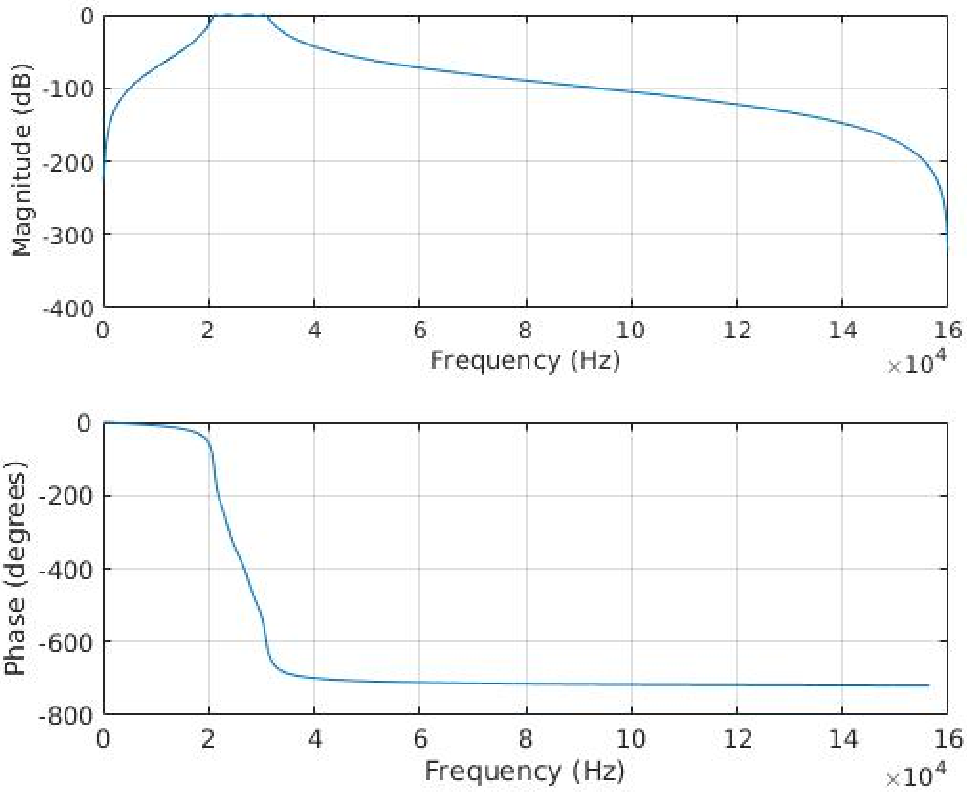

Figure 2: Plotting frequency and phase response of filter using freqz command of MATLAB

2.9 Realization using Direct Form II

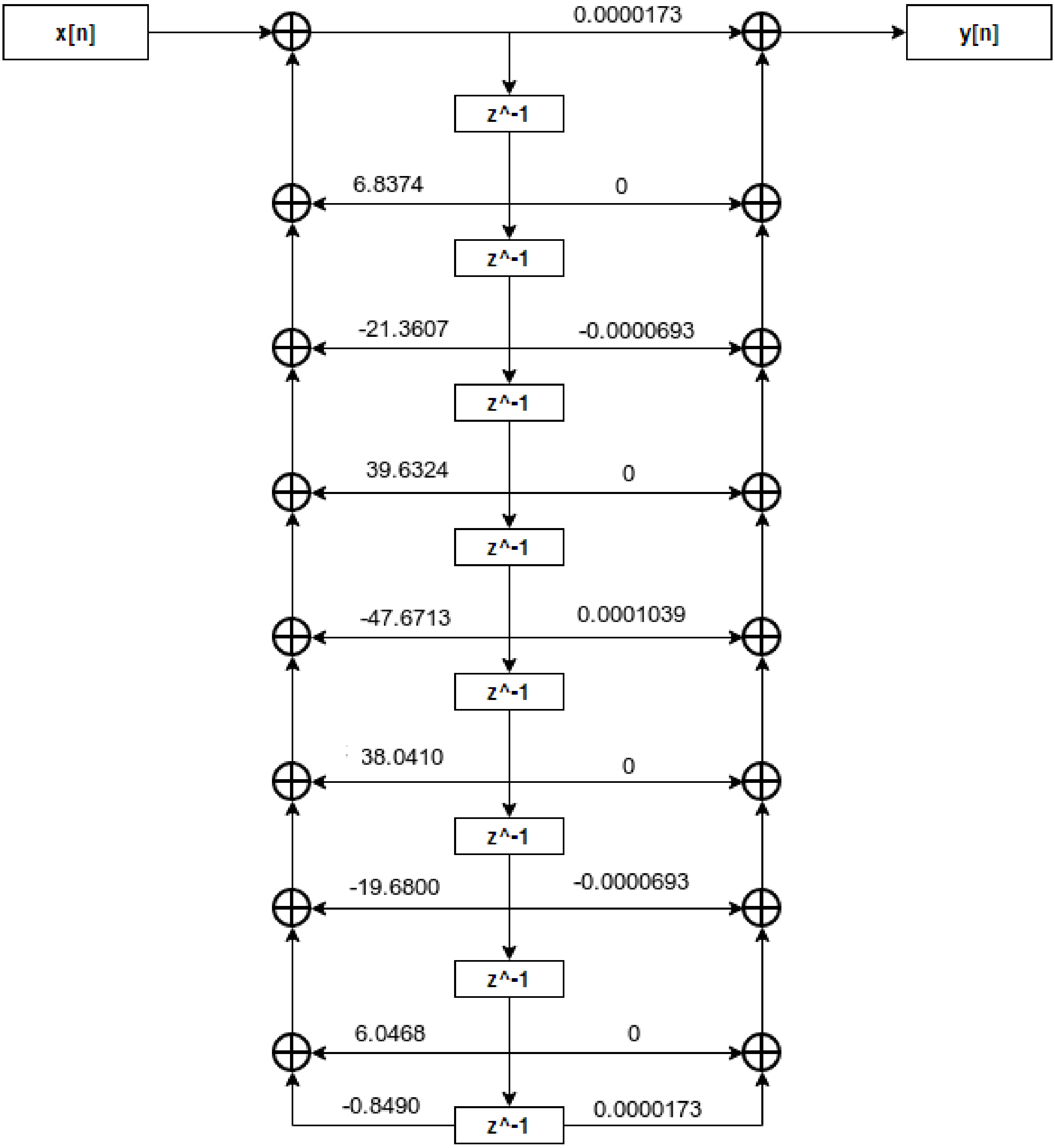

Figure 3: Direct Form II Block Diagram for $H_{discrete,BPF}(z)$

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

2.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be:

$$A = -20\log(0.15) = 16.4782dB$$

Since A $<$ 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, the lower bound on the window length is found using the empirical formula.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width, which is same on both sides of the passband.

$$\Delta\omega_T = \frac{2kHz * 2\pi}{320kHz} = 0.0125\pi$$

$$\therefore N \geq 95$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB, it was found that a window length of **131** is required to satisfy the required constraints.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. The band-pass impulse response samples were generated as the difference between two low-pass filters as done in class.

```
Columns 1 through 15

-0.0019  0.0029  0.0067  0.0085  0.0079  0.0053  0.0018  -0.0013  -0.0032  -0.0033  -0.0022  -0.0007  0.0002  -0.0003  -0.0021

Columns 16 through 30

-0.0045  -0.0063  -0.0063  -0.0038  0.0008  0.0065  0.0115  0.0139  0.0126  0.0075  -0.0000  -0.0079  -0.0139  -0.0161  -0.0140

Columns 31 through 45

-0.0084  -0.0011  0.0055  0.0095  0.0100  0.0075  0.0037  0.0006  -0.0003  0.0015  0.0049  0.0078  0.0079  0.0035  -0.0051

Columns 46 through 60

-0.0159  -0.0253  -0.0293  -0.0250  -0.0117  0.0081  0.0294  0.0459  0.0518  0.0437  0.0225  -0.0073  -0.0378  -0.0606  -0.0687

Columns 61 through 75

-0.0588  -0.0328  0.0029  0.0388  0.0653  0.0750  0.0653  0.0388  0.0029  -0.0328  -0.0588  -0.0687  -0.0606  -0.0378  -0.0073

Columns 76 through 90

0.0225  0.0437  0.0518  0.0459  0.0294  0.0081  -0.0117  -0.0250  -0.0293  -0.0253  -0.0159  -0.0051  0.0035  0.0079  0.0078

Columns 91 through 105

0.0049  0.0015  -0.0003  0.0006  0.0037  0.0075  0.0100  0.0095  0.0055  -0.0011  -0.0084  -0.0140  -0.0161  -0.0139  -0.0079

Columns 106 through 120

-0.0000  0.0075  0.0126  0.0139  0.0115  0.0065  0.0008  -0.0038  -0.0063  -0.0063  -0.0045  -0.0021  -0.0003  0.0002  -0.0007

Columns 121 through 131

-0.0022  -0.0033  -0.0032  -0.0013  0.0018  0.0053  0.0079  0.0085  0.0067  0.0029  -0.0019
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Figure 4: Time domain sequence values

The z-transform can simply be read off from the sequence values since its finite sequence.

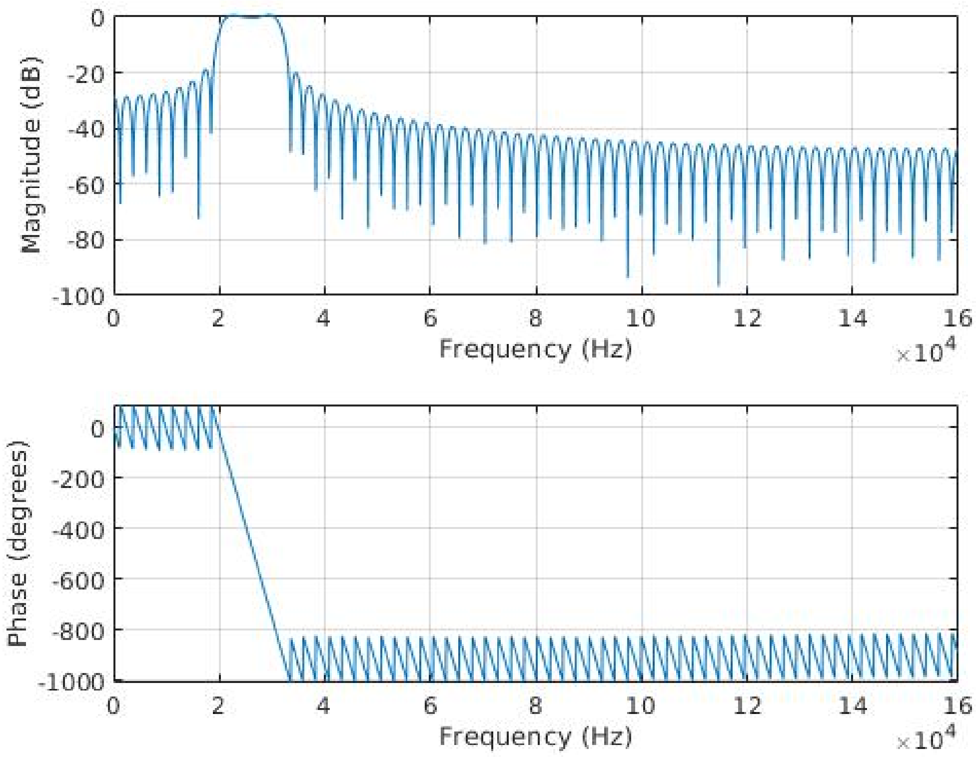

Figure 5: Plotting frequency and phase response of filter using freqz command of MATLAB

3 Filter-2(Bandstop) Details

3.1 Un-normalized Discrete Time Filter Specifications

Filter Number = 79
Since filter number is >75 , m = 79 - 75 = 4 and passband will be monotonic
q(m) = greatest integer strictly less than 0.1*m = greatest integer strictly less than 0.4 = 0
r(m) = m - 10*q(m) = 4 - 10*0 = 4
B_L(m) = 5 + 1.2*q(m) + 2.5*r(m) = 5 + 1.2*0 + 2.5*4 = 15
B_H(m) = B_L(m) + 6 = 15 + 6 = 21

The second filter is given to be a ***Band-Stop*** filter with stopband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are:

- **Stopband** : 15 kHz to 21 kHz
- **Transition Band** : 2 kHz on either side of stopband
- **Passband** : 0-13 kHz and 23-125 kHz (∵ Sampling rate is 250 kHz)

- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.2 Normalized Digital Filter Specifications

Sampling Rate = 250 kHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 125 kHz($\frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as:

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(SamplingRate)}$$

Therefore the corresponding normalized discrete filter specifications are:

- **Stopband** : 0.12π to 0.168π
- **Transition Band** : 0.016π on either side of stopband
- **Passband** : 0-0.104π and 0.184π-π
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.3 Analog filter specifications for Band-stop analog filter using Bilinear Transformation

The bilinear transformation is given as:

$$\Omega = tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get:

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are:

- **Stopband** : 0.1908(Ω_{S_1}) to 0.2702(Ω_{S_2})
- **Transition Band** : 0.1684 to 0.1908 & 0.2702 to 0.2794
- **Passband** : 0 to 0.1684(Ω_{P_1}) and 0.2974(Ω_{P_2}) to ∞
- **Tolerance** : 0.15 in magnitude for both Passband and Stopband

ω	Ω
0	0
0.104π	0.1648
0.12π	0.1908
0.168π	0.2702
0.184π	0.2974
π	∞

- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.4 Frequency Transformation & Relevant Parameters

We need to transform a Band-Stop analog filter to a Lowpass analog filter. We require two parameters and can use the bandstop transformation.

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations:

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.1648 * 0.2974} = 0.2214$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 0.2974 - 0.1648 = 0.1325$$

Ω	Ω_L
0^+	0^+
$0.1648(\Omega_{P_1})$	$+1(\Omega_{L_{P_1}})$
$0.1908(\Omega_{S_1})$	$+2.0026(\Omega_{L_{S_1}})$
$0.2214(\Omega_0^-)$	∞
$0.2214(\Omega_0^+)$	-∞
$0.2702(\Omega_{S_2})$	-1.4924$(\Omega_{L_{S_2}})$
$0.2794(\Omega_{P_2})$	-1$(\Omega_{L_{P_2}})$
∞	0^-

3.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge** : 1 (Ω_{L_P})
- **Stopband Edge** : min$(\Omega_{L_{S_1}}$,-$\Omega_{L_{S_2}})$ = min(2.0026, 1.4924) = 1.4924 (Ω_{L_S})

- **Tolerance** : 0.15 in magnitude for both Passband and Stopband
- **Passband Nature** : Monotonic
- **Stopband Nature** : Monotonic

3.6 Analog Lowpass Transfer Function

We need an Analog Filter which has a *monotonic passband* and a *monotonic stopband*. Therefore we need to design using the **Butterworth** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, we define two new quantities in the following way:

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$

$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now using the inequality on the order N of the filter for the Butterworth Approximation we get:

$$N_{min} = \lceil \frac{\log \sqrt{\frac{D_2}{D_1}}}{\log \frac{\Omega_S}{\Omega_P}} \rceil$$

$$N_{min} = \lceil 5.9046 \rceil = 6$$

The cut-off frequency(Ω_c) of the Analog LPF should satisfy the following constraint:

$$\frac{\Omega_P}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_S}{D_2^{\frac{1}{2N}}}$$

$$1.083 \leq \Omega_c \leq 1.0899$$

Thus we can choose the value of Ω_c to be 1.085
Now, the poles of the transfer function can be obtained by solving the equation:

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.085}\right)^{12} = 0$$

Solving for the roots (using MATLAB) we get: Note that the above figure shows the poles of the Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$p_1 = -0.2808 - 1.0480\iota$$

$$p_2 = -0.7672 - 0.7672\iota$$

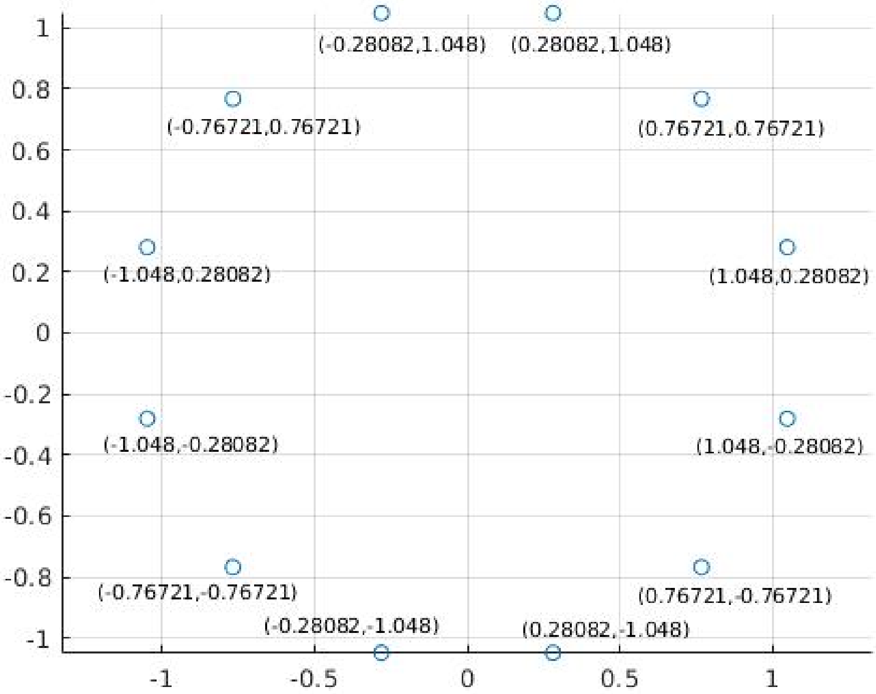

Figure 6: Poles of Magnitude Plot of Analog LPF

$$p_3 = -1.0480 - 0.2808\iota$$
$$p_4 = -1.0480 + 0.2808\iota$$
$$p_5 = -0.7672 + 0.7672\iota$$
$$p_6 = -0.2808 + 1.0480\iota$$

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as:

$$H_{analog,LPF}(s_L) = \frac{(\Omega_c)^N}{(s_L - p_1)(s_L - p_2)(s_L - p_3)(s_L - p_4)(s_L - p_5)(s_L - p_6)}$$

$$= \frac{1.6315}{(s_L^2 + 0.5616s_L + 1.1772)(s_L^2 + 1.5344s_L + 1.1772)(s_L^2 + 2.0961s_L + 1.1772)}$$

3.7 Analog Bandstop Transfer Function

The transformation equation is given by:

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Substituting the values of the parameters B(0.1325) and Ω_0(0.2214), we get:

$$s_L = \frac{0.1325s}{0.0490 + s^2}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BSF}(s)$. It can be written in the form N(s)/D(s) where the coefficients of the polynomials N(s) and D(s) are given as:

Degree	s^{12}	s^{11}	s^{10}	s^9
Coefficient	$1(a_{12})$	$0.4719(a_{11})$	$0.4054(a_{10})$	$0.1323(a_9)$

Degree	s^8	s^7	s^6	s^5
Coefficient	$0.0595(a_8)$	$0.01389(a_7)$	$0.004126(a_6)$	$0.00068(a_5)$

Degree	s^4	s^3	s^2	s^1	s^0
Coefficient	$0.000143(a_4)$	$1.558*10^{-5}(a_3)$	$2.34*10^{-6}(a_2)$	$1.3348*10^{-7}(a_1)$	$1.386*10^{-8}(a_0)$

Table 1: Coefficients of D(s)

Degree	s^{12}	s^{10}	s^{8}	s^{6}
Coefficient	$1(b_{12})$	$0.2941(b_{10})$	$0.0360\ (b_8)$	$0.024(b_6)$

Degree	s^4	s^2	s^0
Coefficient	$8.656*10^{-5}(b_4)$	$1.697*10^{-6}(b_2)$	$1.3863*10^{-8}(b_0)$

Table 2: Coefficients of N(s)

The coefficients of odd powers of s in N(s) are all 0.

3.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BSF}(z)$ from $H_{analog,BSF}(s)$.It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as:

Degree	z^{-12}	z^{-11}	z^{-10}	z^{-9}
Coefficient	$0.64\ (b_{-12})$	$-6.94(b_{-11})$	$35.3(b_{-10})$	$-110.79(b_{-9})$

Degree	z^{-8}	z^{-7}	z^{-6}	z^{-5}
Coefficient	$238.9\ (b_{-8})$	$-372.68(b_{-7})$	$431.15(b_{-6})$	$-372.68(b_{-5})$

Degree	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^0
Coefficient	$238.9(b_{-4})$	$-110.79(b_{-3})$	$35.3(b_{-2})$	$-6.94(b_{-1})$	$0.64(b_0)$

Table 3: Coefficients of N(z)

Degree	z^{-12}	z^{-11}	z^{-10}	z^{-9}
Coefficient	$0.41(a_{-12})$	$-4.76(a_{-11})$	$26.03(a_{-10})$	$-87.86(a_{-9})$

Degree	z^{-8}	z^{-7}	z^{-6}	z^{-5}
Coefficient	$203.91(a_{-8})$	$-342.53(a_{-7})$	$426.97(a_{-6})$	$-397.89(a_{-5})$

Degree	z^{-4}	z^{-3}	z^{-2}	z^{-1}	z^{0}
Coefficient	$275.14(a_{-4})$	$-137.71(a_{-3})$	$47.38(a_{-2})$	$-10.07(a_{-1})$	$1(a_0)$

Table 4: Coefficients of D(z)

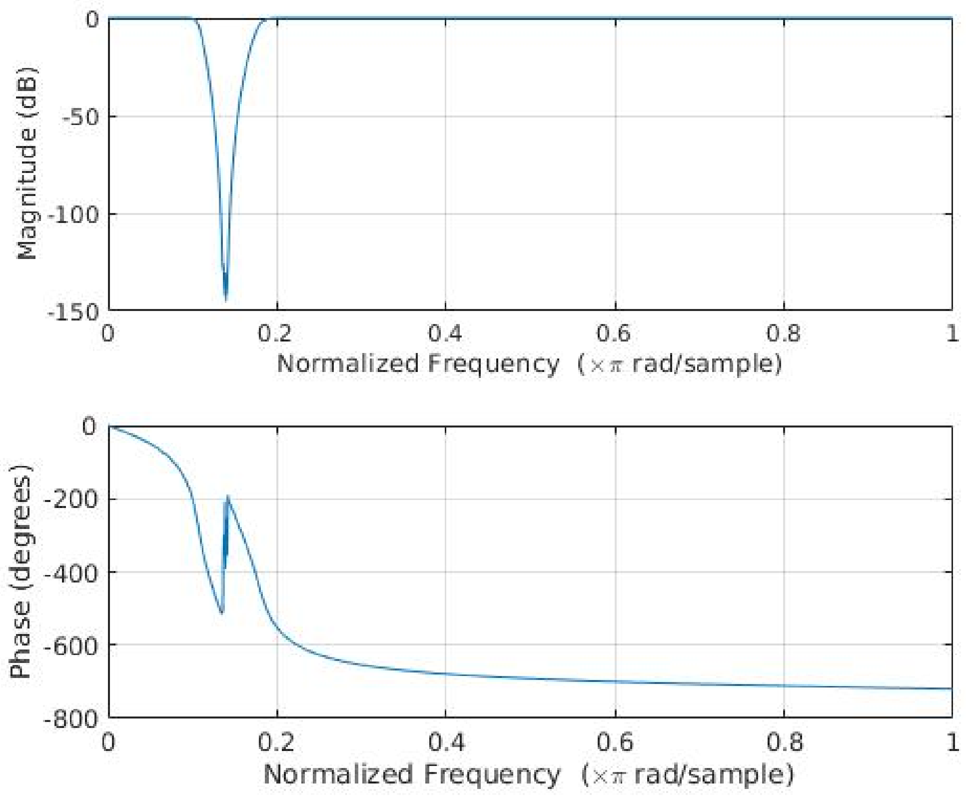

Figure 7: Plotting frequency and phase response of filter using freqz command of MATLAB

3.9 Realization using Direct Form II

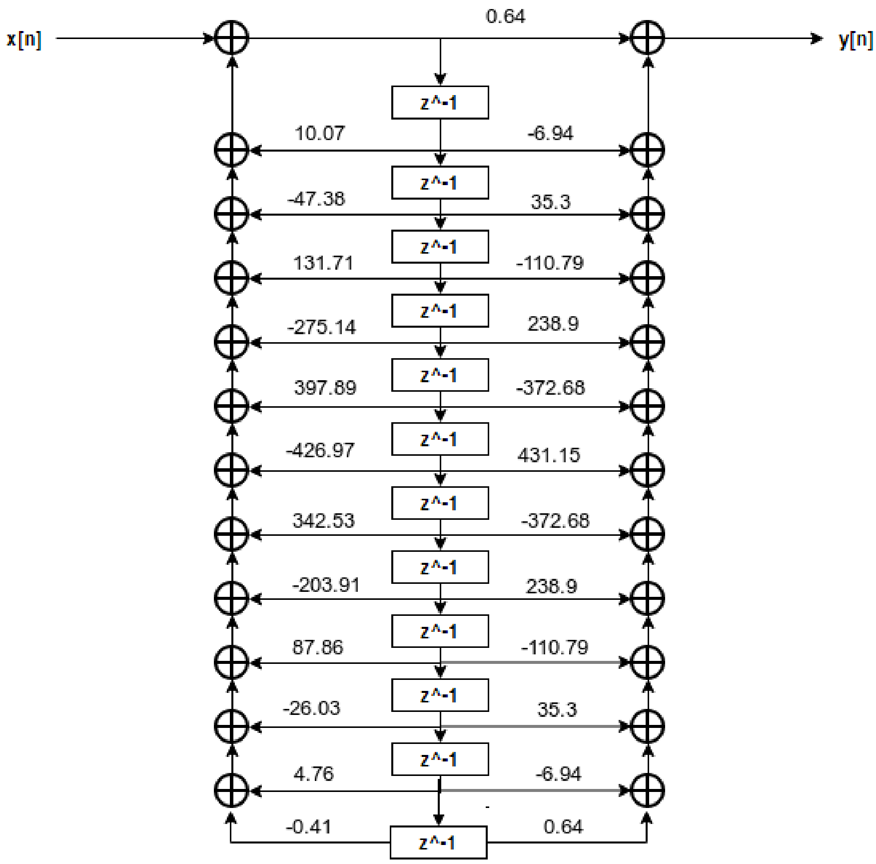

Figure 8: Direct Form II Block Diagram for $H_{discrete,BSF}(z)$

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

3.10 FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15.
Therefore $\delta = 0.15$ and we get the minimum stopband attenuation to be:

$$A = -20\log(0.15) = 16.4782dB$$

Since A $<$ 21, we get β to be 0 where β is the shape parameter of Kaiser window.
Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \geq \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here $\Delta\omega_T$ is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{2kHz * 2\pi}{250kHz} = 0.016\pi$$

$$\therefore N \geq 74$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB,it was found that a window length of **99** is required to satisfy the required constraints. Also, since β is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. The band-stop impulse response samples were generated as the difference between three low-pass filters(all-pass - bandpass) as done in class.

```
Columns 1 through 15

 -0.0125  -0.0127  -0.0101  -0.0052   0.0009   0.0068   0.0113   0.0132   0.0123   0.0089   0.0041  -0.0009  -0.0048  -0.0068  -0.0066

Columns 16 through 30

 -0.0048  -0.0024  -0.0005  -0.0001  -0.0014  -0.0042  -0.0073  -0.0092  -0.0086  -0.0046   0.0024   0.0114   0.0200   0.0259   0.0268

Columns 31 through 45

  0.0213   0.0098  -0.0060  -0.0230  -0.0371  -0.0448  -0.0435  -0.0325  -0.0135   0.0101   0.0333   0.0509   0.0588   0.0548   0.0391

Columns 46 through 60

  0.0147  -0.0134  -0.0393  -0.0575   0.9360  -0.0575  -0.0393  -0.0134   0.0147   0.0391   0.0548   0.0588   0.0509   0.0333   0.0101

Columns 61 through 75

 -0.0135  -0.0325  -0.0435  -0.0448  -0.0371  -0.0230  -0.0060   0.0098   0.0213   0.0268   0.0259   0.0200   0.0114   0.0024  -0.0046

Columns 76 through 90

 -0.0086  -0.0092  -0.0073  -0.0042  -0.0014  -0.0001  -0.0005  -0.0024  -0.0048  -0.0066  -0.0068  -0.0048  -0.0009   0.0041   0.0089

Columns 91 through 99

  0.0123   0.0132   0.0113   0.0068   0.0009  -0.0052  -0.0101  -0.0127  -0.0125
```

Figure 9: Time domain sequence values

The z-transform can simply be read off from the sequence values since its finite sequence.

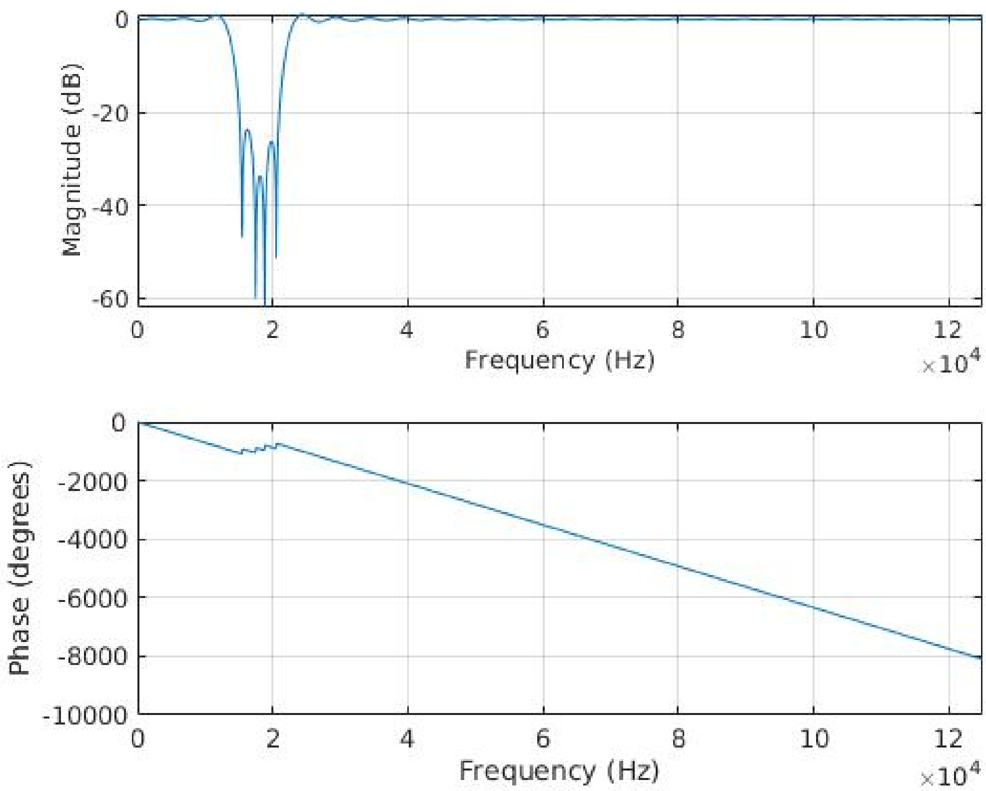

Figure 10: Plotting frequency and phase response of filter using freqz command of MATLAB

4 MATLAB Plots

4.1 Filter 1 - Bandpass

4.1.1 IIR Filter

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

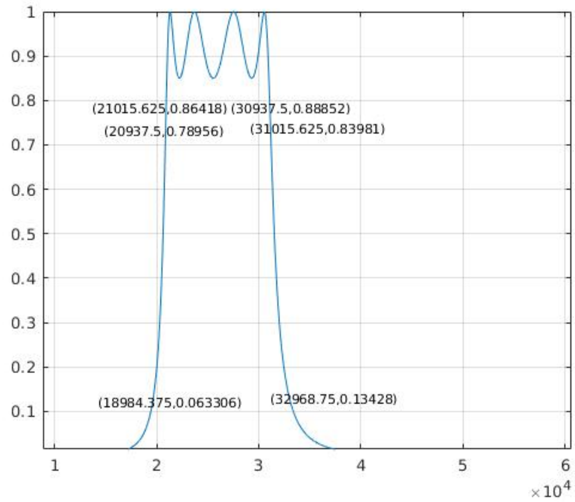

Figure 11: Frequency Response

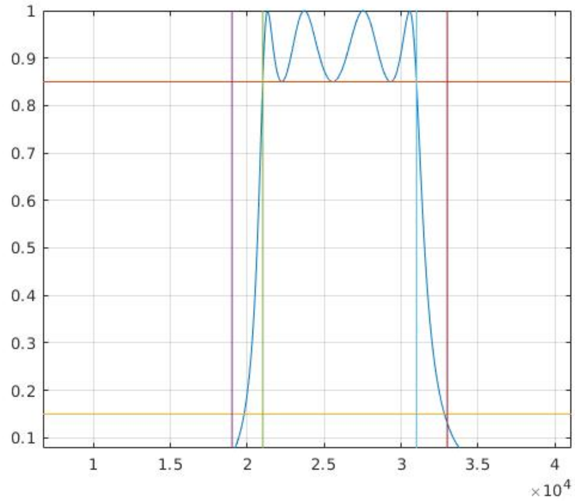

Figure 12: Frequency Response with boundaries

It can be seen that the **phase response** is **not linear**.

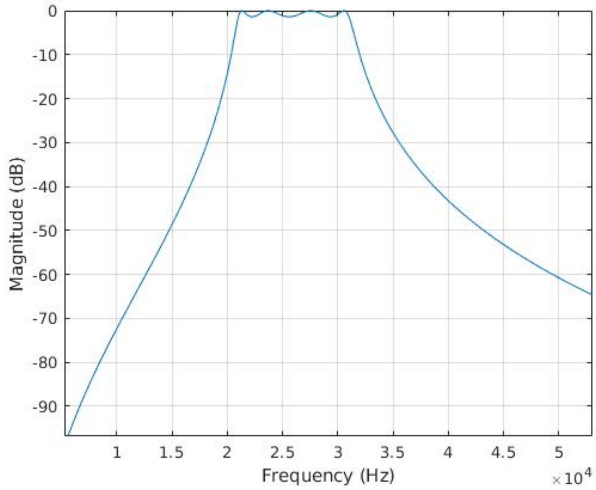

Figure 13: Frequency Response (in dB)

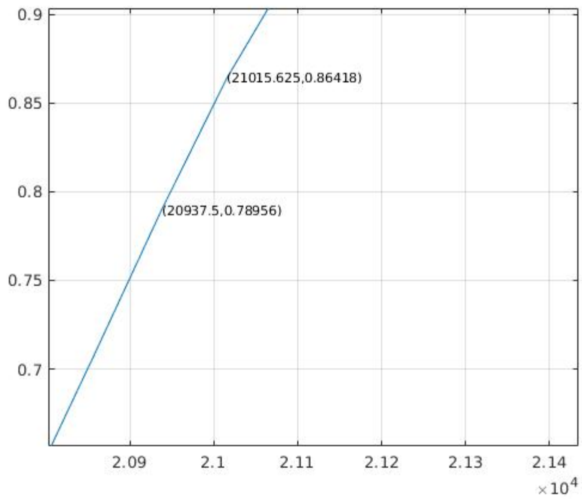

Figure 14: Frequency Response- Around Passband Limit 1

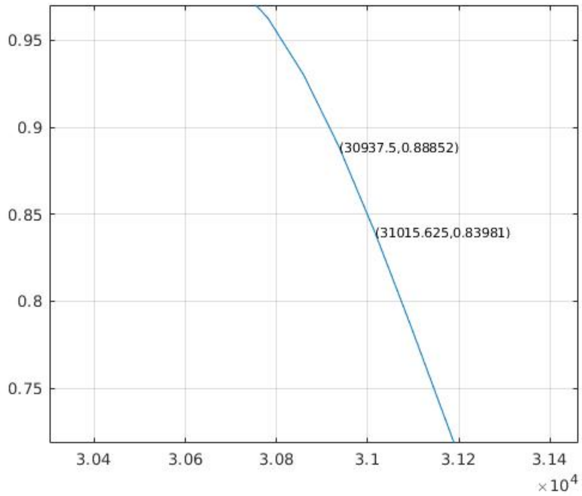

Figure 15: Frequency Response- Around Passband Limit 2

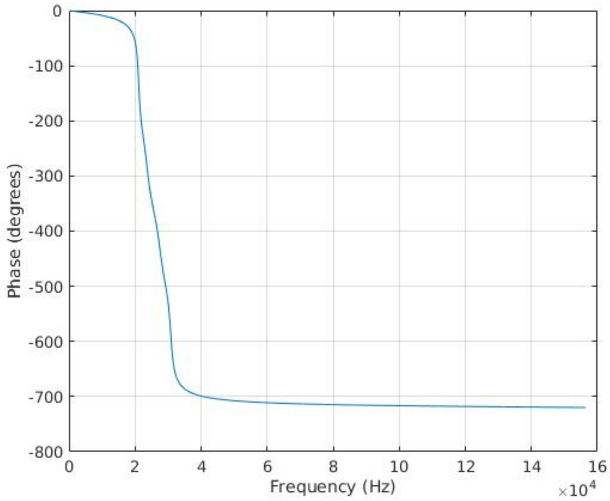

Figure 16: Phase Response

4.1.2 FIR Filter

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter is indeed giving us a **Linear Phase** response which is desired.

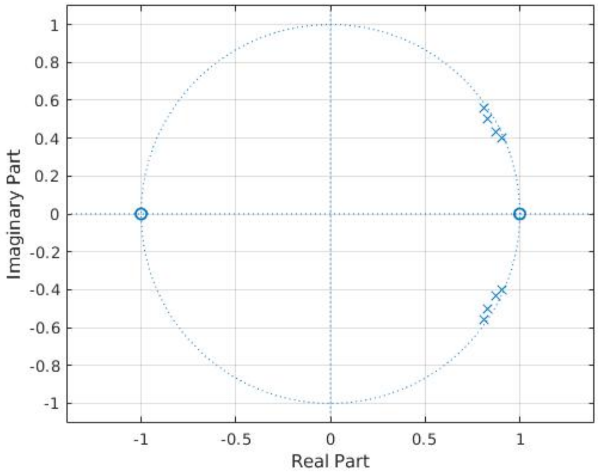

Figure 17: Pole-Zero map (all poles within unit circle, hence stable)

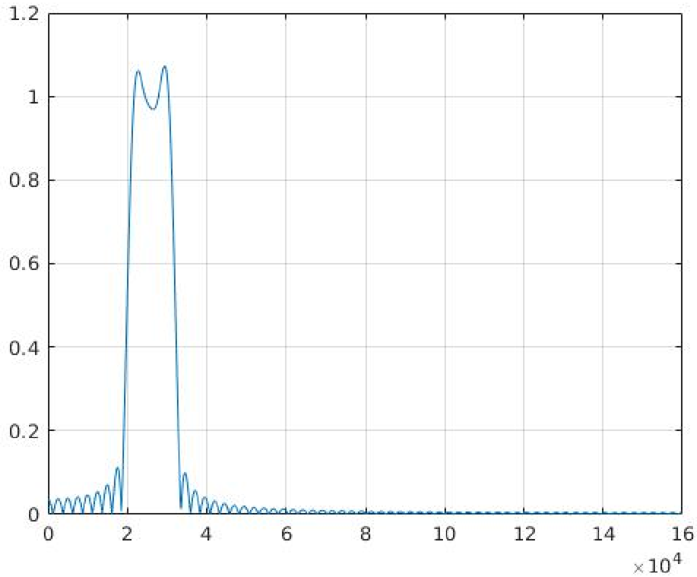

Figure 18: Frequency Response

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

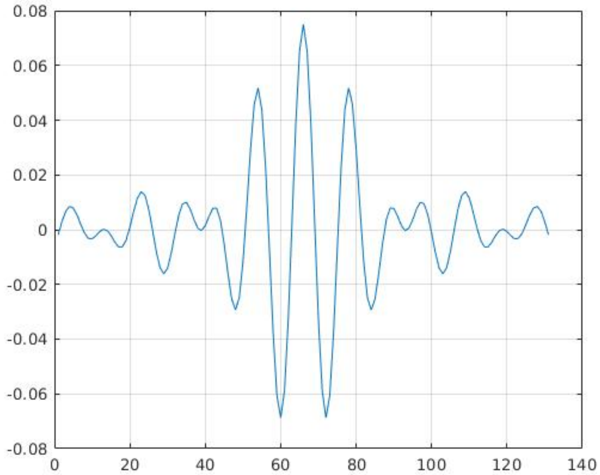

Figure 19: Time Domain Sequence

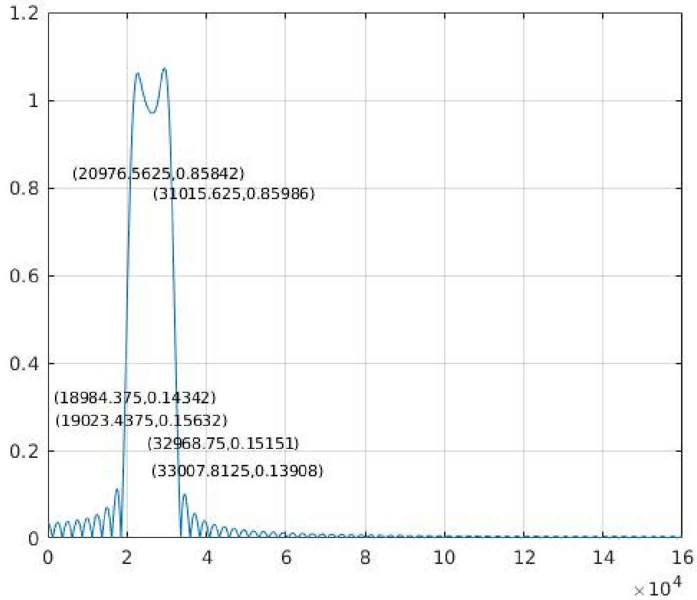

Figure 20: Magnitude Plot

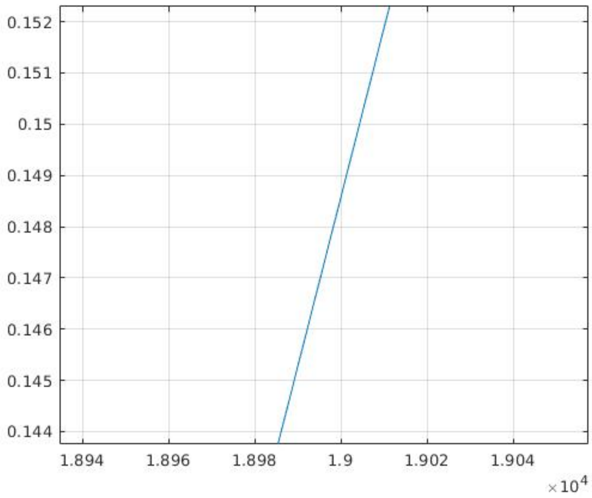

Figure 21: Frequency Response- Around Stopband Limit 1

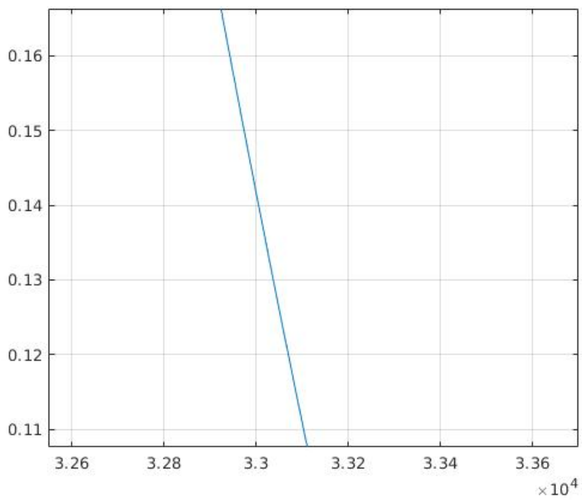

Figure 22: Frequency Response- Around Stopband Limit 2

4.2 Filter 2 - Bandstop

4.2.1 IIR Filter

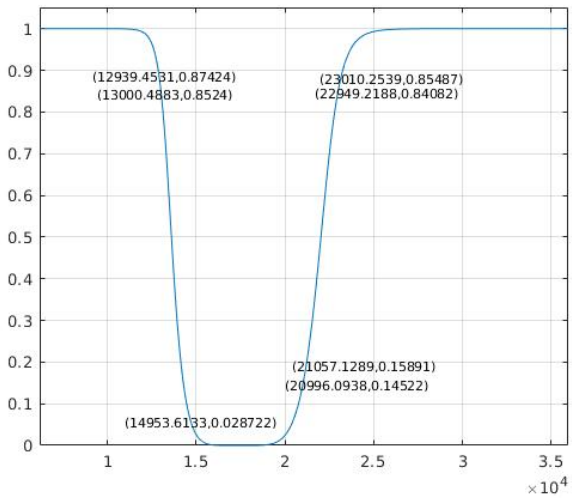

Figure 23: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. In the above plot, the band edge frequencies have been marked. From the magnitude at

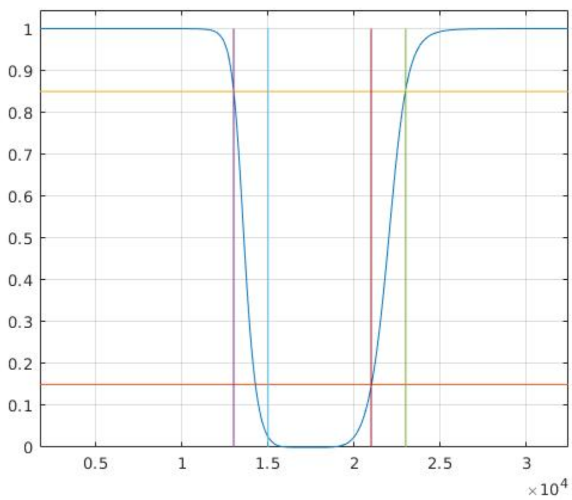

Figure 24: Frequency Response with boundaries

these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

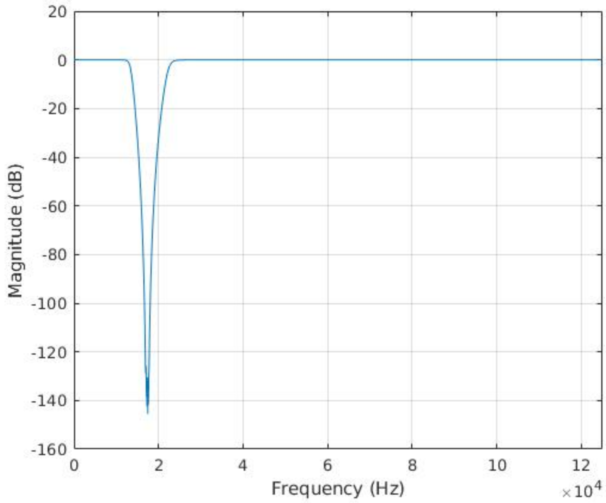

Figure 25: Frequency Response (in dB)

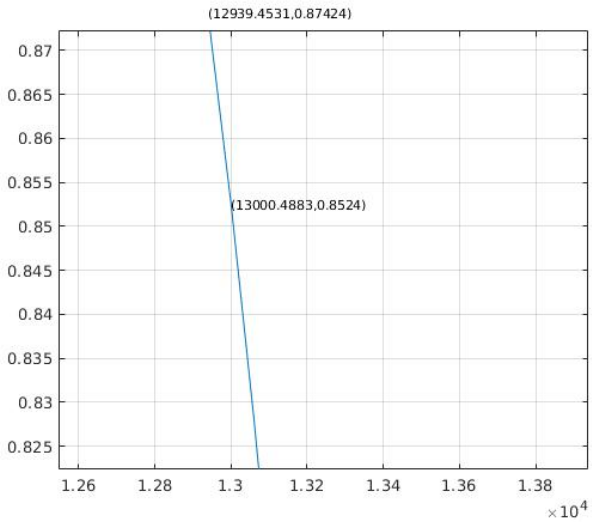

Figure 26: Frequency Response- Around Passband Limit 1

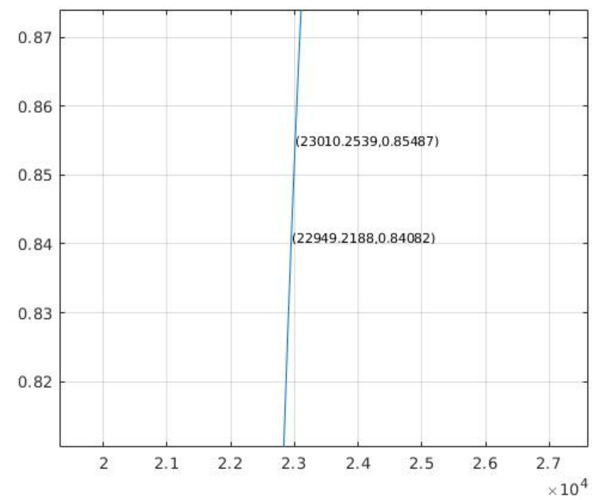

Figure 27: Frequency Response- Around Passband Limit 2

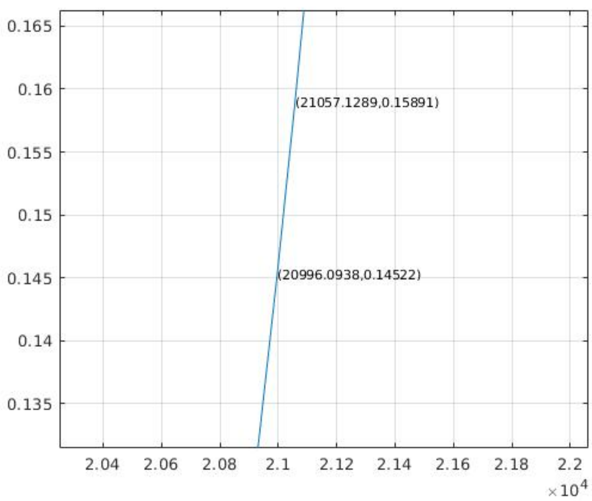

Figure 28: Frequency Response- Around Stopband Limit 2

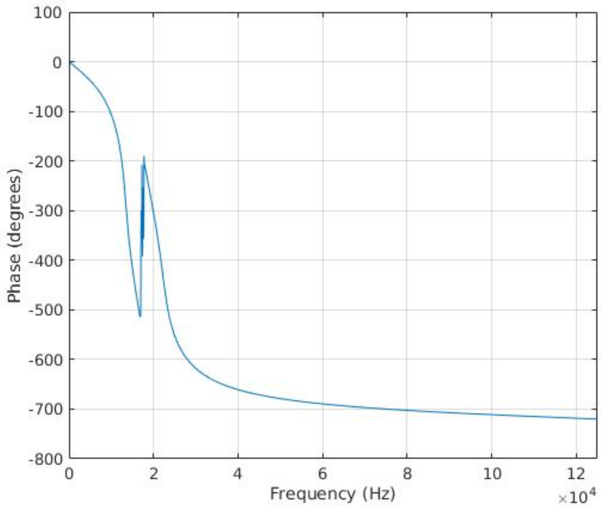

Figure 29: Phase Response

It can be seen that the **phase response** is **not linear**.

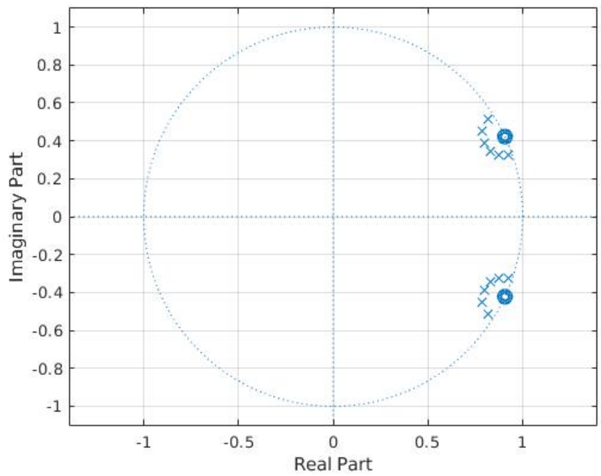

Figure 30: Pole-Zero map (all poles within unit circle, hence stable)

4.2.2 FIR Filter

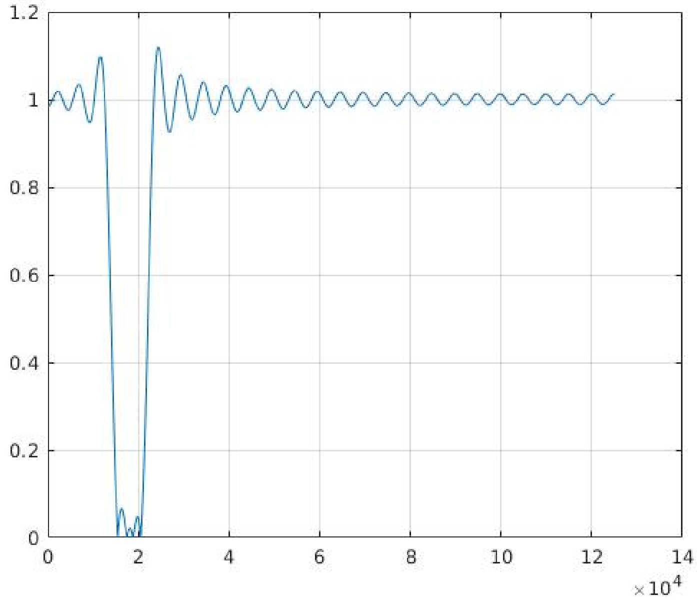

Figure 31: Frequency Response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter indeed gives us a **Linear Phase** response which is desired.

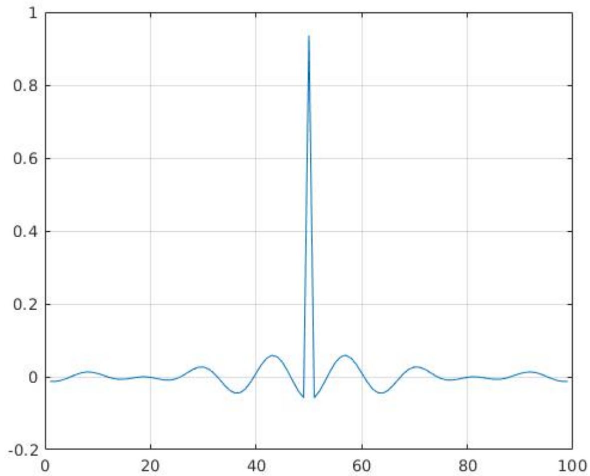

Figure 32: Time Domain Sequence

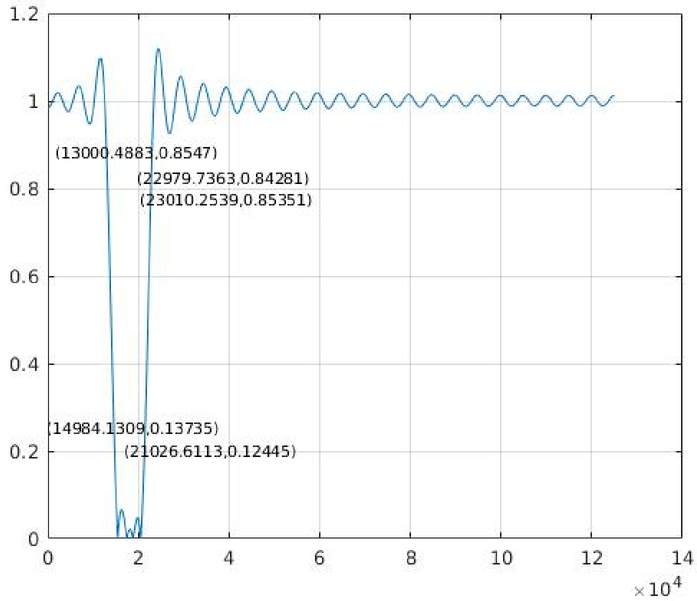

Figure 33: Magnitude Plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

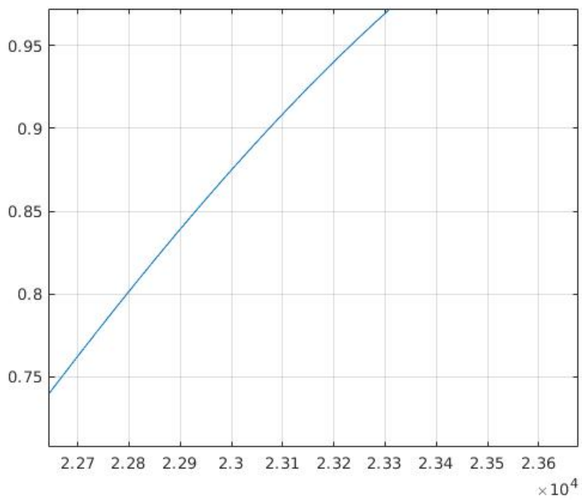

Figure 34: Frequency Response- Around Passband Limit 2